FSM decomposition

Now, we will discuss a very simple model for FSM decomposition. As an example, we use Mealy FSM S_6 (Table 1) and a partition π on the set of its states:

$$\pi = \{A_1, A_2, A_3\};$$
$$A_1 = \{a_2, a_3, a_9\};\ A_2 = \{a_4, a_7, a_8\};\ A_3 = \{a_1, a_5, a_6\}.$$

The number of component FSMs in the FSM network is equal to the number of blocks in partition π. Thus, in our example, we have three component FSMs S^1, S^2, S^3.

Let B^m is the set of states in the component FSM S^m. B^m contains the corresponding block of the partition π plus one additional state b_m. So, in our example:

S^1 has the set of states $B^1 = \{a_2, a_3, a_9, b_1\}$;
S^2 has the set of states $B^2 = \{a_4, a_7, a_8, b_2\}$;
S^3 has the set of states $B^3 = \{a_1, a_5, a_6, b_3\}$.

Table 1. Mealy FSM S_6

a_m	a_s	$X(a_m,a_s)$	$Y(a_m,a_s)$	H
a1	a3	x1*x2*x3	y1y2	1
a1	a6	x1*x2*~x3	y2y12	2
a1	a1	x1*~x2	y1y2	3
a1	a5	~x1	y1y2y12	4
a2	a2	x6	--	5
a2	a3	~x6	y3y5	6
a3	a3	x10	y3y5	7
a3	a9	~x10*x4	y10y15	8
a3	a8	~x10*~x4	y5y8y9	9
a4	a6	x7	y13	10
a4	a4	~x7*x9	y13y18	11
a4	a8	~x7*~x9	y13y14	12
a5	a6	x1	y16y17	13
a5	a5	~x1	y7y11	14
a6	a1	x5	y1y2	15
a6	a1	~x5	y16y17	16
a7	a2	x8	y14y18	17
a7	a4	~x8	y13y18	18
a8	a7	x9	y4y6	19
a8	a4	~x9	y6	20
a9	a9	x11*x6	y10y15	21
a9	a2	x11*~x6	y5y8y9	22
a9	a3	~x11	y3y8y9	23

To construct a transition table for each component FSM we should define the transitions between the states of these FSMs. For this, each transition between two states a_i and a_j of Mealy FSM S_6 from Table 1 should be implemented one after another as one or two transitions in component FSMs. Two possible cases are available:

1. In Mealy FSM S_6, there is a transition between a_i and a_j (Fig. 1, left) and both these states are in the same component FSM S^m. In such a case, we have the same transition in this component FSM S^m (Fig. 21, right). It means that we must rewrite the corresponding row from the table of FSM S_6 into the table of component FSM S^m.

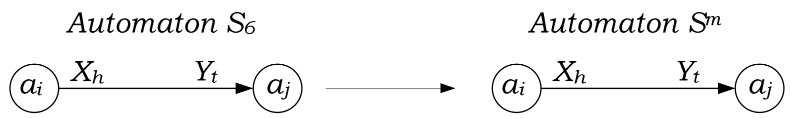

Figure 1. Two states a_i and a_j are in the same component FSM

2. Two states a_i and a_j are in different component FSMs (Fig. 2). Let a_i is in the component FSM S^m ($a_i \in B^m$) and a_j is in the component FSM S^p ($a_j \in B^p$). In such a case, one transition of FSM S_6 should be presented as two transitions – one in the component FSM S^m and one in the component FSM S^p:

   - FSM S^m transits from a_i into its additional state b_m with the same input X_h. At its output, we have the same output variables from set Y_t plus one additional output variable z_j, where index j is the index of state a_j in the component FSM S^p.
   - FSM S^p is in its additional state b_p. It transits from this state into state a_j with input signal z_j, that is an additional output variable in the component FSM S^m. The output at this transition is Y_0 – the signal with all output variables equal to zero.

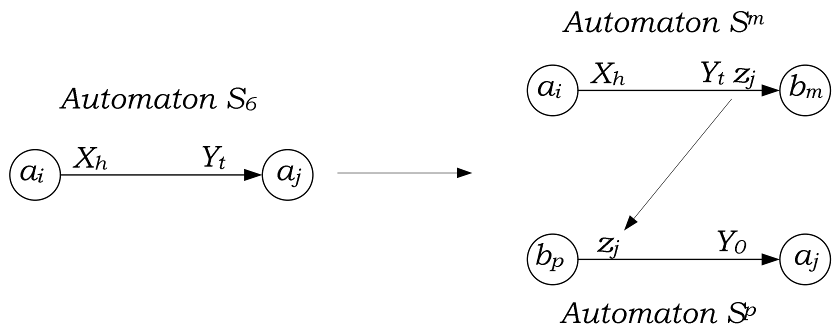

Figure 2. Two states a_i and a_j are in the different component FSMs

Thus, the procedure for FSM decomposition is reduced to:

a) Copying the row

$$a_i \quad a_j \quad X(a_i, a_j) \quad Y(a_i, a_j)$$

from the table of the decomposed FSM S to the table of the component FSM S^m if both states a_i and a_j are the states of S^m;

b) Replacement the row

$$a_i \quad a_j \quad X(a_i, a_j) \quad Y(a_i, a_j)$$

in the table of the decomposed FSM S by the row

$$a_i \quad b_m \quad X(a_i, a_j) \quad Y(a_i, a_j)\ z_j$$

in the table of the component FSM S^m, and by the row

$$b_p \quad a_j \quad z_j \quad \text{--}$$

in the table of the component FSM S^p, if a_i is the state of S^m and a_j is the state of S^p.

As a result of decomposition of FSM S_6, we obtain the network with three component FSMs in Fig. 3. Their transition tables are presented in Tables 2 – 4.

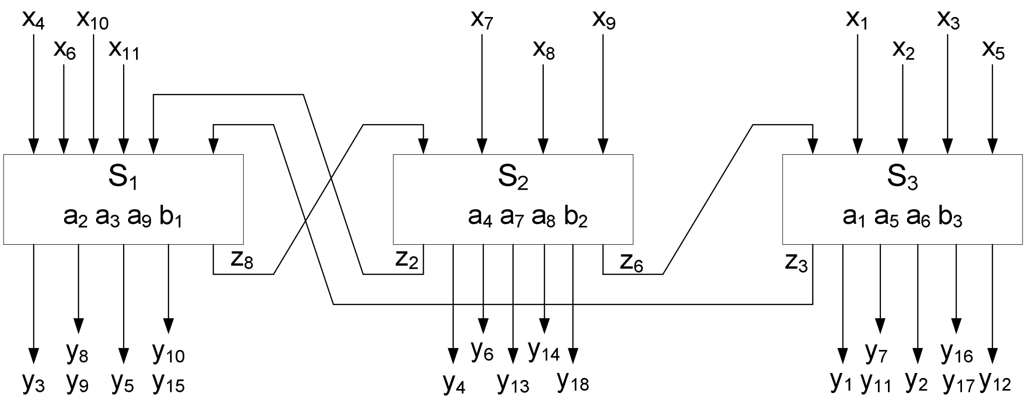

Figure 3. Network with three component FSMs

Table 2. Component FSM S^1

a_m	a_s	$X(a_m, a_s)$	$Y(a_m, a_s)$	H
a2	a2	x6	--	1
a2	a3	~x6	y3y5	2
a3	a3	x10	y3y5	3
a3	a9	~x10*x4	y10y15	4
a3	b1	~x10*~x4	y5y8y9z8	5
a9	a9	x11*x6	y10y15	6
a9	a2	x11*~x6	y5y8y9	7
a9	a3	~x11	y3y8y9	8
b1	a2	z2	--	9
b1	a3	z3	--	10
b1	b1	~z2*~z3	--	11

Table 3. Component FSM S^2

a_m	a_s	$X(a_m,a_s)$	$Y(a_m,a_s)$	H
a4	b2	x7	y13z6	1
a4	a4	~x7*x9	y13y18	2
a4	a8	~x7*~x9	y13y14	3
a7	b2	x8	y14y18z2	4
a7	a4	~x8	y13y18	5
a8	a7	x9	y4y6	6
a8	a4	~x9	y6	7
b2	a8	z8	--	8
b2	b2	~z8	--	9

Table 4. Component FSM S^3

a_m	a_s	$X(a_m,a_s)$	$Y(a_m,a_s)$	H
a1	b3	x1*x2*x3	y1y2z3	1
a1	a6	x1*x2*~x3	y2y12	2
a1	a1	x1*~x2	y1y2	3
a1	a5	~x1	y1y2y12	4
a5	a6	x1	y16y17	5
a5	a5	~x1	y7y11	6
a6	a1	x5	y1y2	7
a6	a1	~x5	y16y17	8
b3	a6	z6	--	9
b3	b3	~z6	--	10

Now we will illustrate some examples of transitions for cases (a) and (b):

a. In FSM S_6, there is a transition from state *a2* to state *a3* with input *~x6* and output *y3y5* (row 6 in Table 1). As both these states *a2* and *a3* are in the same component FSM S_1, in this FSM there is a transition from *a2* to *a3* with the same input *~x6* and the same output *y3y5* (row 2 in Table 2). Exactly in the same way, we rewrite row 12 of Table 1 to row 3 of Table 3 and row 2 of Table 1 to row 2 of Table 4 because the current states and the next states are in the same component FSMs.

b. In FSM S_6, there is a transition from state *a3* to state *a8* with input *~x10*~x4* and the output *y5y8y9* (row 9 in Table 1). Since *a3* is the state of component FSM S^1 and *a8* is the state of another component FSM S^2, in FSM S^1 there is a transition from *a3* to *b1* with the same input *~x10*~x4* and output *y5y8y9z8* (row 5 in Table 2). The last output *z8* is the input of FSM S^2 that wakes this FSM and transits it from state *b2* to state *a8* (row 8 in Table 3). Similarly, we convert row 1 of Table 1 into two rows – the first in Table 4 and the tenth in Table 2 etc. Note that we add the last row in each FSM table to remain component FSMs in the state b_m if each z_j is equal to zero.

Now we will discuss how this network works. Let a_1 is an initial state in FSM S_6. After decomposition, state a_1 is in FSM S^3, so, at the beginning, just FSM S^3 is in state *a1.* Other FSMs are in states *b1* and *b2* correspondingly, it is possible to say that they "are sleeping" in these states. FSM S^3 transits from the state to the state until *x1*x2*x3 = 1* in state *a1* (see row 1 in Table 4). Only at this transition FSM S^3 produces output signal *z3* and transits into state *b3* (sleeping state). This signal *z3* is the input signal of FSM S^1. It wakes FSM S^1 and transits it from the sleeping state *b1* to state *a3* (see row 10 in Table 2). Now FSM S^1 transits from the state to the state until, in state *a3,* it transits into state *b1* with input signal *~x10*~x4 = 1* and wakes FSM S^2 by signal *z8* (see row 5 in Table 2 and row 8 in Table 3).

Unlike FSMs S^1 and S^3, the component FSM S^2 has two possibility to wake other component FSMs – in state *a4* with input signal *x7 = 1* (row 1 of Table 3) and in state *a7* with input signal *x8 = 1* (row 4 in the same Table), etc. Thus, in each time, all component FSMs, except one, are in the states of type b_m and only one of them is in the state of type a_i..